A Strategy for STEM Learning in a Changing Society: Focusing on the Undergraduate Program

Mathematics Education Using Real-World Problems

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Background

- It is increasingly important for *all students* at all educational levels to acquire *mathematical skills for use in real-world situations*.

Mathematical Literacy

- An important issue in Japan: *mathematical literacy education* for *'Bunkei'* students (=humanities & social sciences students)
  - Aims of mathematical literacy educations for *Bunkei* students:
    - to foster students' *mathematical skill for use in real-world situations*
    - to improve students' *attitude towards math*
  - However, it is challenging because many of those students have
    - *math anxiety* and *difficulty* in learning math,
    - *little knowledge* about *how mathematics is used* in the real world.
- ***Question: How should we design mathematical literacy education for Bunkei students?***

New Math Courses (2012-)

- Math courses for the humanities & social sciences students of the first year at Osaka Prefecture University.
    - Basic Math I (spring semester)
    - Basic Math II (Autumn semester)
    - 90 min/week for 14 weeks followed by an examination period
    - 4 classes taught by 4 teachers (60-80 students in each class)

Design Principles

(Kawazoe et al., 2013; Kawazoe & Okamoto, 2017)

1. Design lessons according to *mathematical modelling processes*.
2. Choose *topics & contexts* by considering which mathematical knowledge students are likely to *encounter in real life* and in which *situation* they will encounter it.
3. *Present problems in different contexts* associated with the same mathematical knowledge.
4. *Connect different mathematical knowledge together* by using different mathematizations of the same problem or mutually related contexts.
5. Explain mathematical concepts & operations using *both mathematical language & everyday language*.
6. Engage students in *group activities* rather than individual ones.
7. Design worksheets based on analysis of *students' understanding process* and use them as *tools for formative assessment*.

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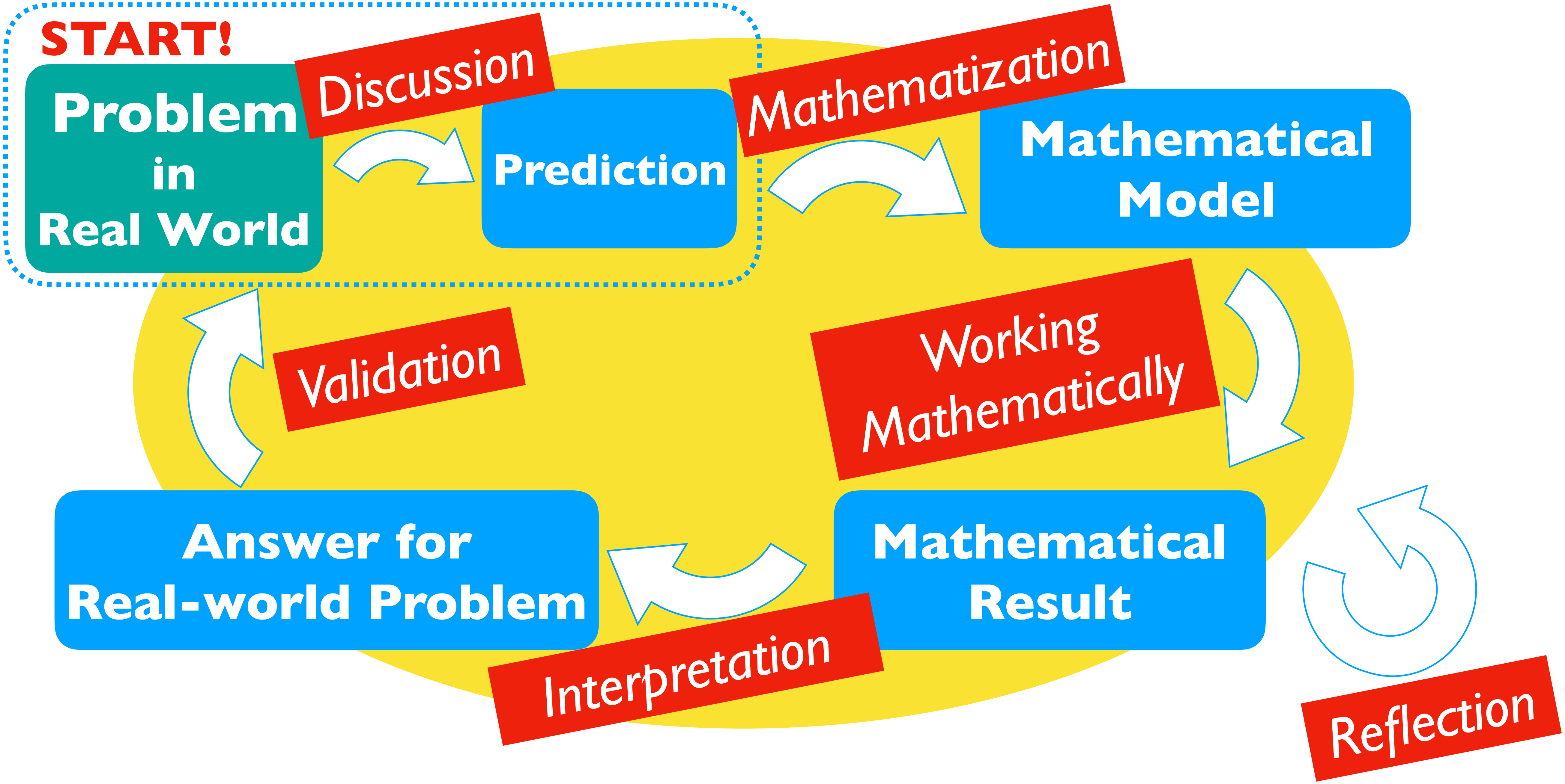

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Basic Math I (Spring semester)

- systems of linear equations/inequalities (linear programming)
- number sequences (savings, loan payment, pharmacokinetics, model of addiction, etc.)
- matrices & vectors (spreadsheets, social networks analysis)
- functions (mental rotation, pharmacokinetics, bacterial growth, pandemic, etc.)
- probability (lottery, disease examination, birthday paradox, Bayesian estimation)

Basic Math II (Autumn semester)

- eigenvalues/vectors (population dynamics, optimal distribution, etc.)
- functions (cyclical movement of electric demand, sound composition, etc.)
- derivatives (innovation diffusion, population growth, logistic function, marginal profit)
- integrals (speed & distance, accumulated radiation level, standard normal distribution)
- multivariable functions (loan simulator, formulas for estimating vital capacity, optimization problem, etc.)

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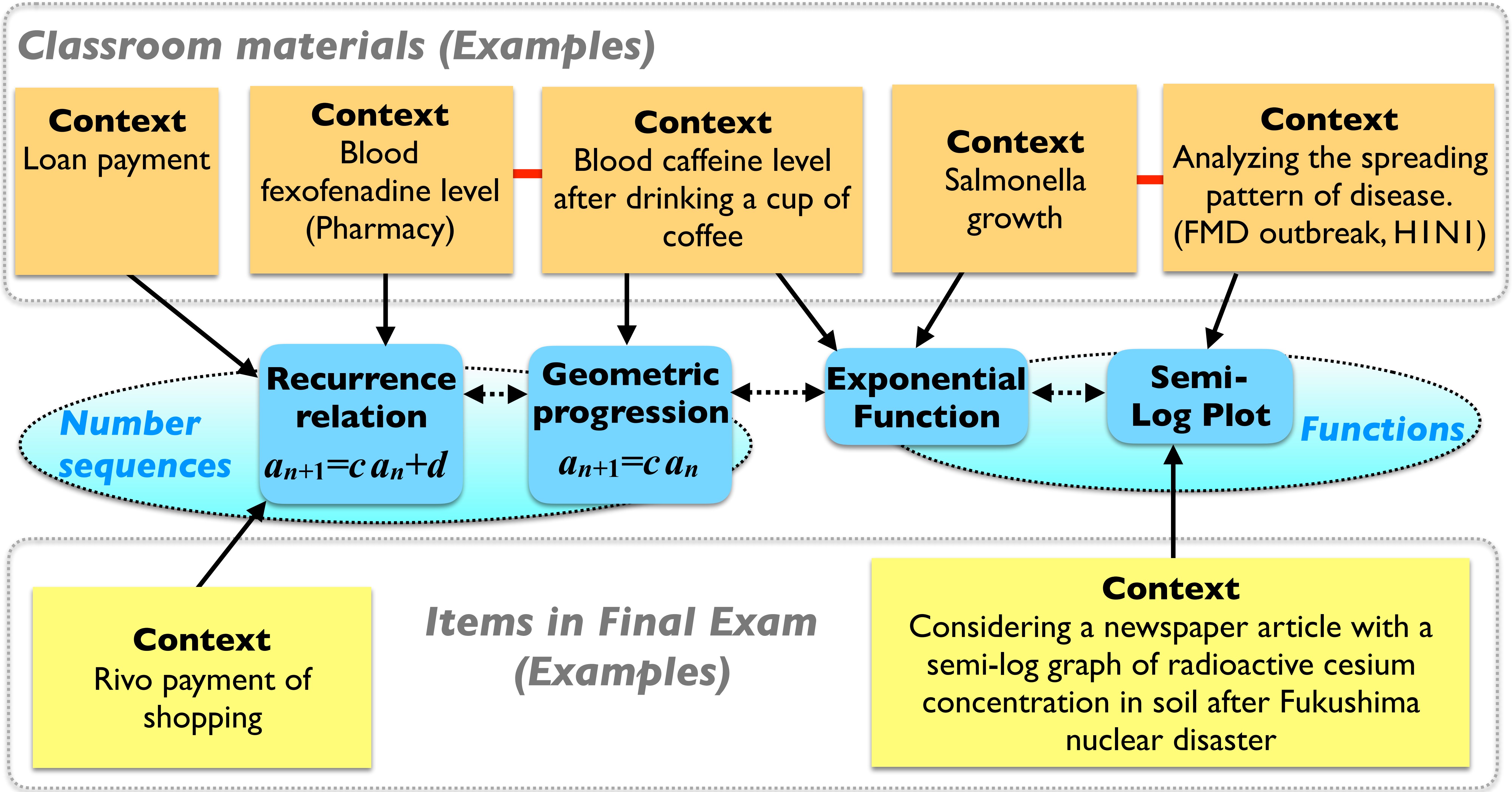

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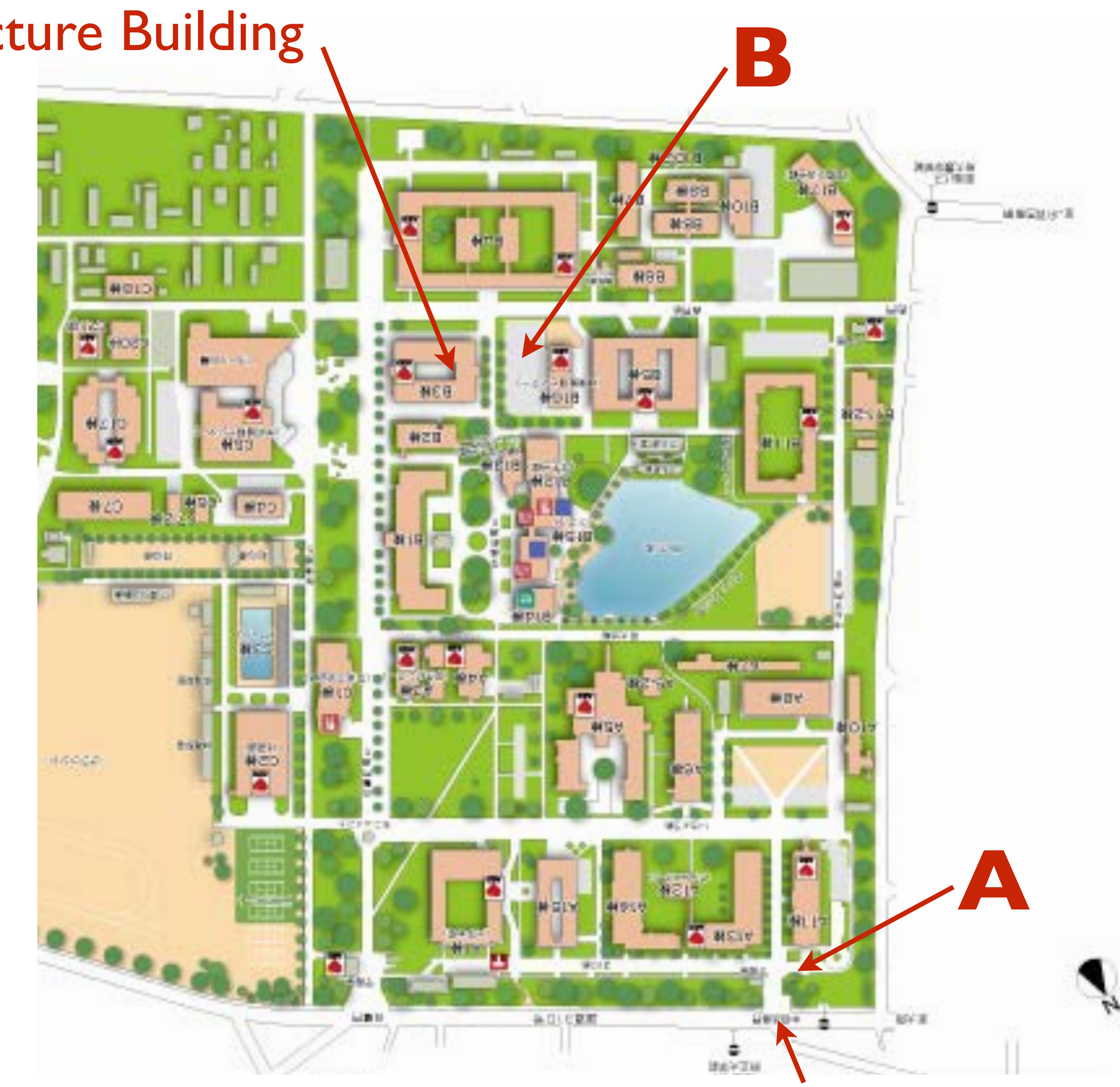
Example: Eigenvalues/vectors (2-weeks lesson)

Rental Bicycle Service Problem

- 2 Parkings (A, B)
- Bicycles can be dropped off at any of the 2 parkings
- Monitoring investigation (weekly data)
- The service will start with more than 100 bicycles.
- How to divide bicycles between 2 parkings?

Students already learned the transition matrix, but have not learned eigenvectors/values yet.



From \ To	A	B
A	70	30
B	20	80

(%)

1st week: discovering eigenvectors

- How to divide bicycles between the 2 parkings? *(Group discussion)*

  1. A>B, 2. A=B, 3. A<B

From \ To	A	B
A	70	30
B	20	80

(%)

Simulation! What occurs?

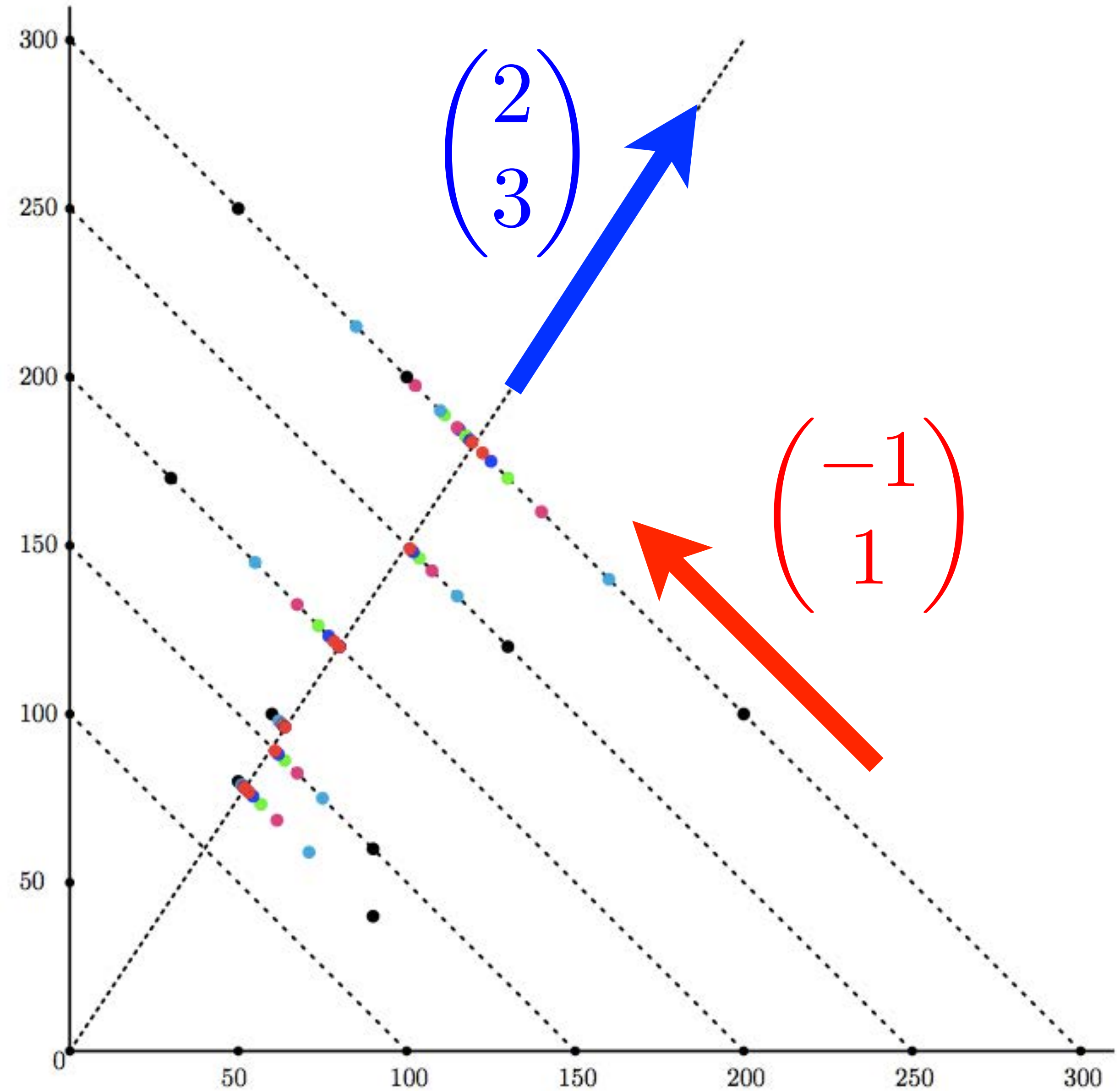
- Choose an initial data and calculate the distribution of the following 5 weeks. *(Group activity)*

$$\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}^n \begin{pmatrix} 200 \\ 100 \end{pmatrix} = \begin{pmatrix} ??? \\ ??? \end{pmatrix}$$

- Plot the data of all groups on the graph paper. *(Group activity)*

What can be found?

- Eigenvectors/values are introduced.

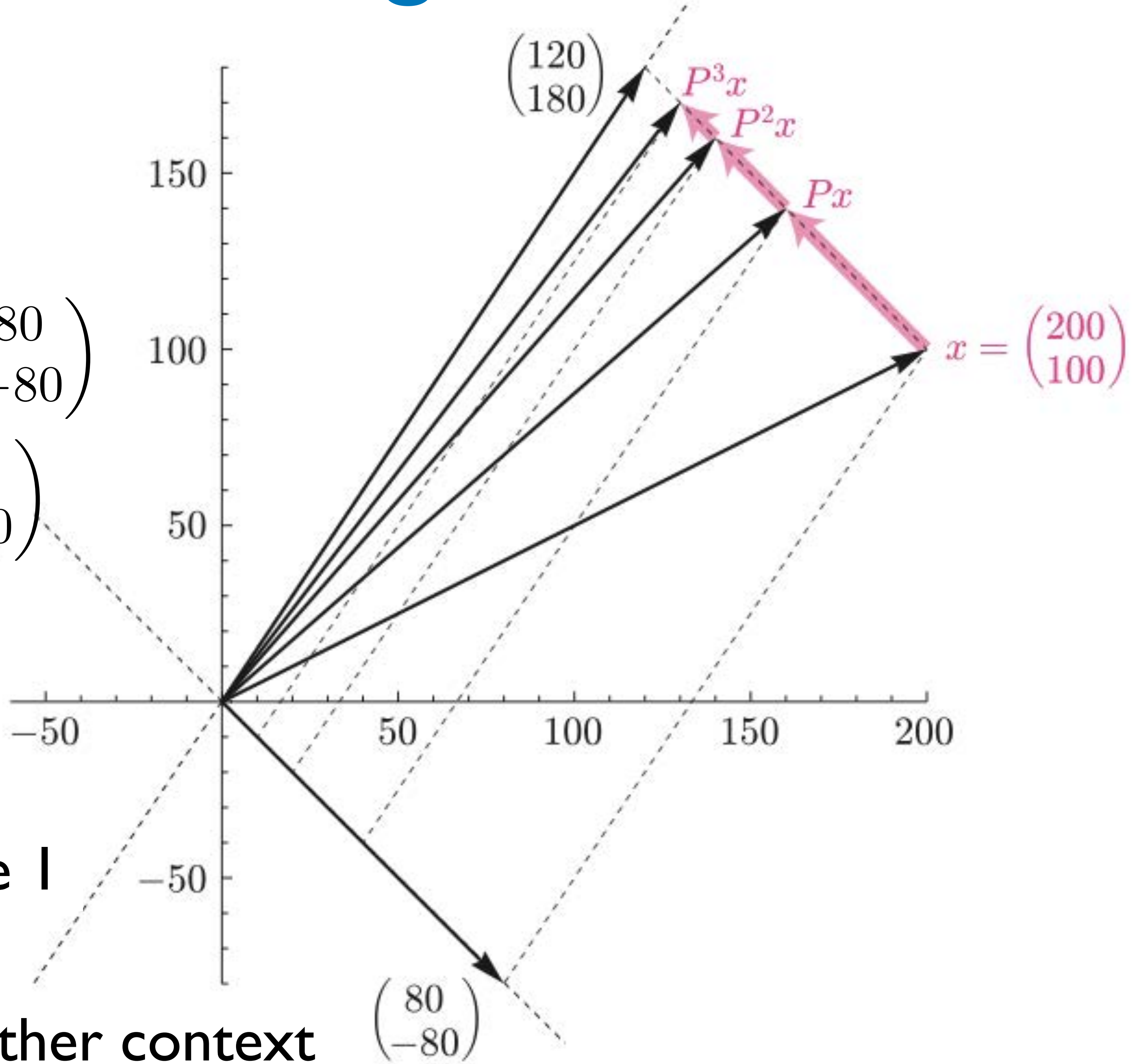


2nd week: explain with eigenvectors

- Re-interpretation:

$$\begin{pmatrix}200\\100\end{pmatrix} =?? \begin{pmatrix}2\\3\end{pmatrix} +?? \begin{pmatrix}-1\\1\end{pmatrix}$$

$$P^n \begin{pmatrix}200\\100\end{pmatrix} = P^n \begin{pmatrix}120\\180\end{pmatrix} + P^n \begin{pmatrix}80\\-80\end{pmatrix}$$
$$= \begin{pmatrix}120\\180\end{pmatrix} + 0.5^n \begin{pmatrix}80\\-80\end{pmatrix}$$



- What is an optimal solution?

  ✓ Optimal solution
  =stable point
  =eigenvector with eigenvalue 1

- Homework:

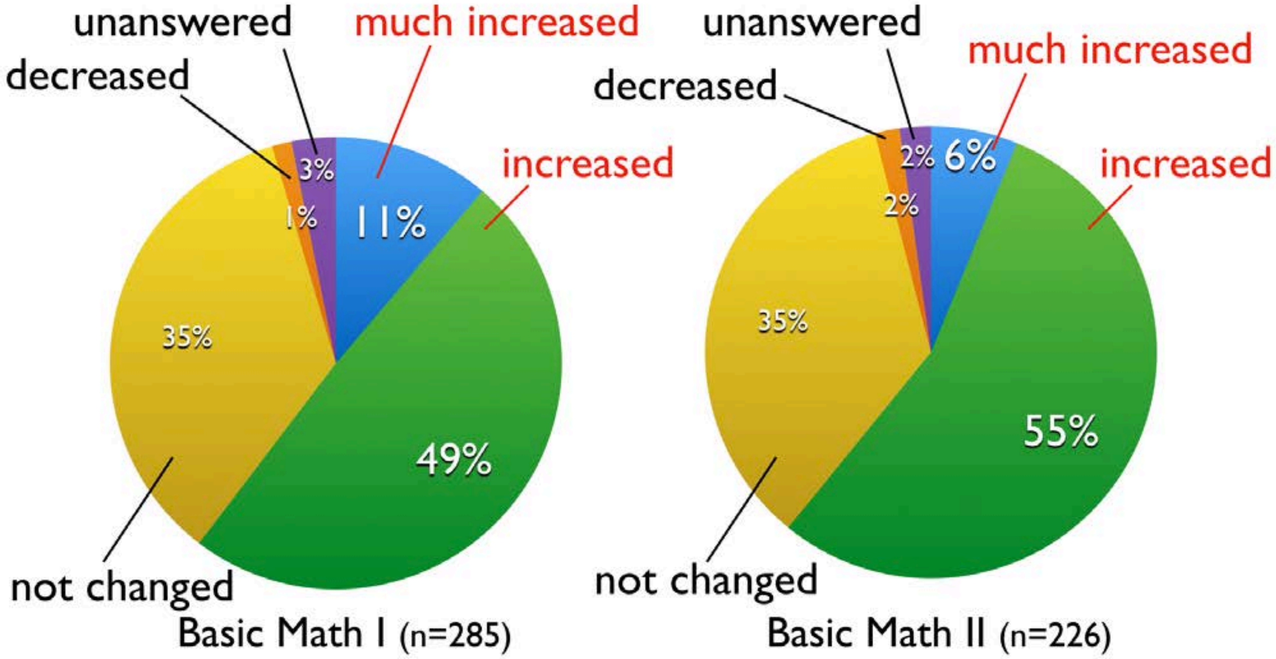
  ✓ a similar problem, but in another context

Evaluation of Teaching Practices

- The average of mean scores of the final exam (2012-2018):
  - 72.3 (BMI) & 74.2 (BMII) (full mark=100)
- Results of a self-report questionnaire (Kawazoe et al., 2013):
  - The rates of students who answered that their *interest in math increased* during the semester were 60.4% (BMI) & 60.6% (BMII).

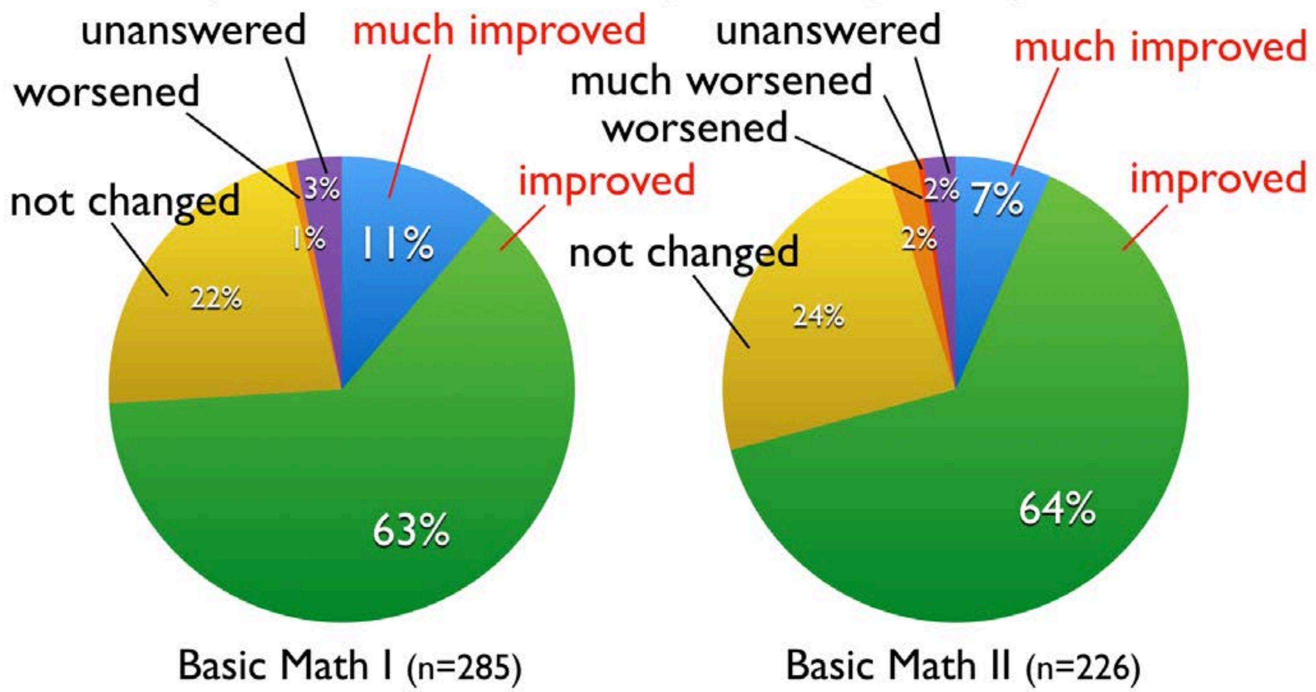
Results of Self-evaluation (1) (Result of 2012 academic year)

"How did your interest on mathematics change during the semester?"



Results of Self-evaluation (2) (Result of 2012 academic year)

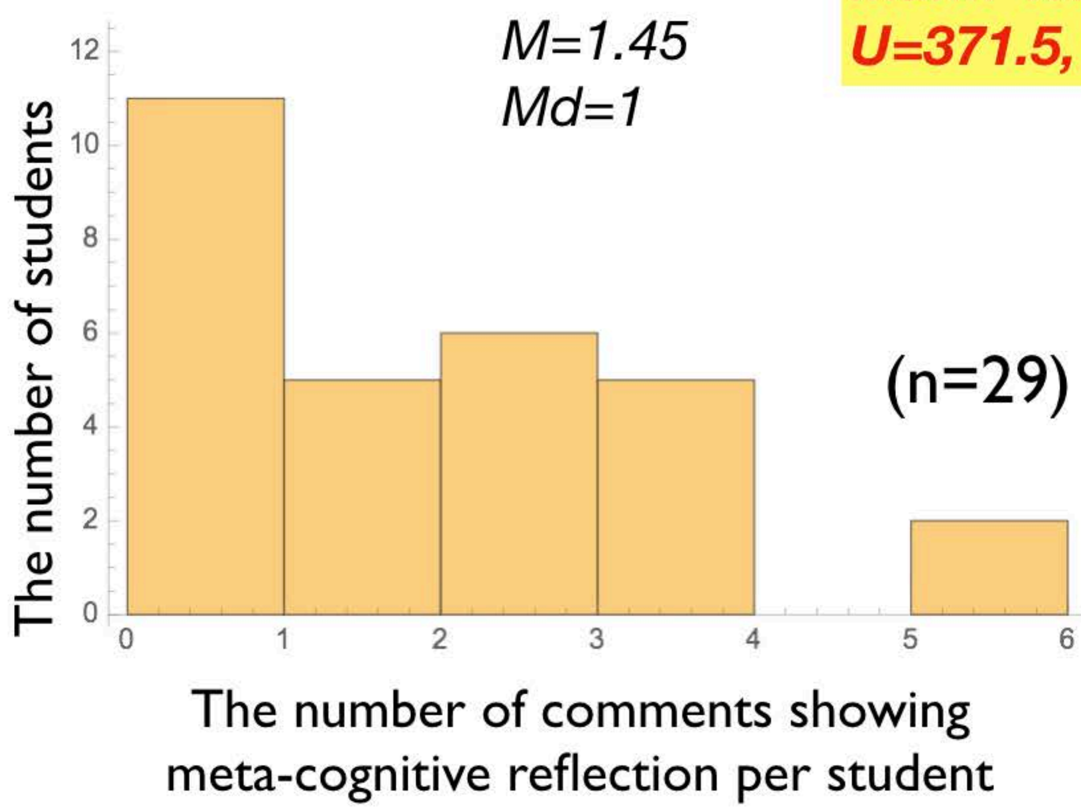
"How did your mathematical thinking skills change during the semester?"



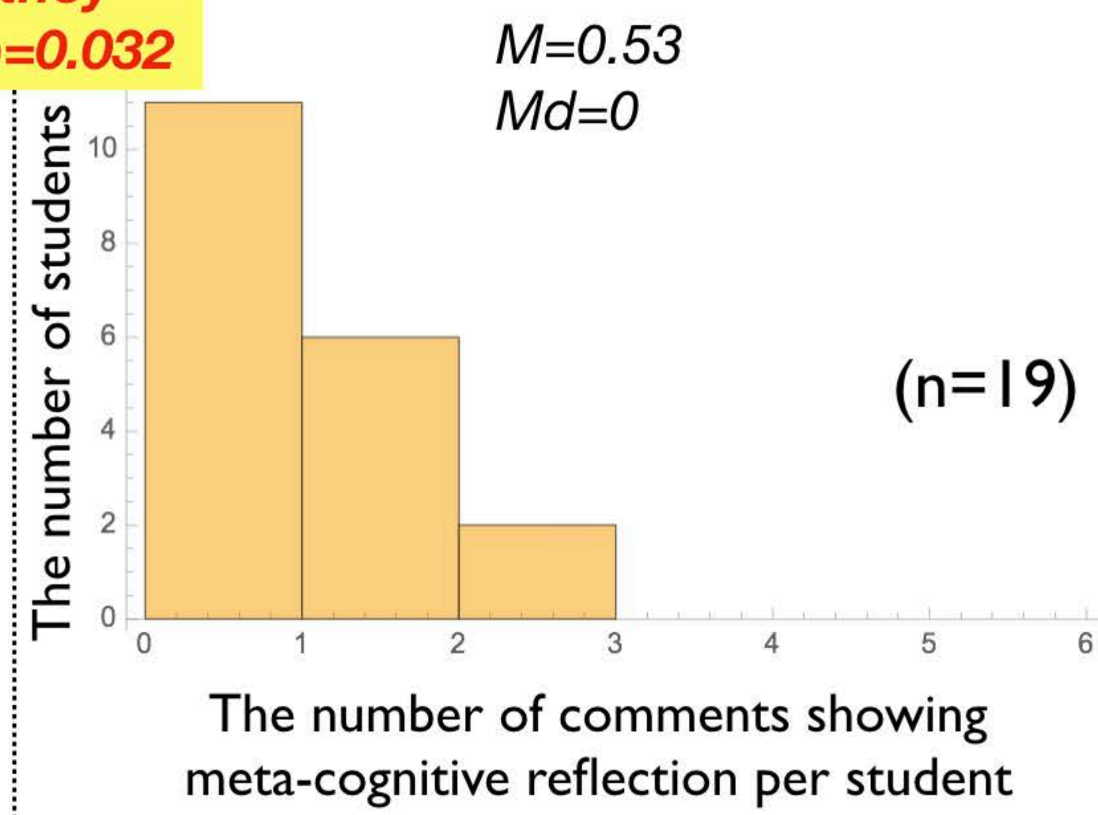
- The rates of students who answered that their *mathematical thinking skills improved* during the semester were 74.0% (BMI) & 70.8% (BMII).

Frequency of meta-cognitive reflections per student

Mann-Whitney
$U=371.5$, $p=0.032$



Student who did not like math



Student who liked math

- Analysis of students' free comments in worksheets (BMI in 2018):
  - The frequency of comments containing descriptions showing *meta-cognitive reflection* per student was *greater for students who did not like mathematics than for students who liked mathematics* at the beginning of the course. (Kawazoe, 2019)

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Thank you.

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